M31

Coimisiún na Scrúduithe Stáit
State Examinations Commission

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Marking Scheme *Leaving Certificate Examination, 2003*

Applied Mathematics *Ordinary Level*

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3), 15 (att 5).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 9 in numerical order, not the order of answering.

5 Scrutinise **all** pages of the answer book.

6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

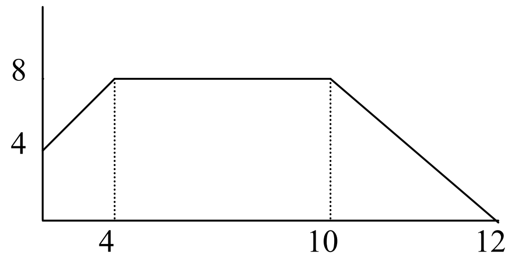
1. A car travels from p to q on a straight level road. It passes p with a speed of 4 m/s and accelerates uniformly to its maximum speed of 8 m/s in 4 seconds. The car maintains this speed of 8 m/s for 6 seconds before decelerating uniformly to rest at q. The car takes 12 seconds to travel from p to q.

(i) Draw a speed-time graph of the motion of the car from p to q.
(ii) Find the uniform acceleration of the car.
(iii) Find the uniform deceleration of the car.
(iv) Find $|pq|$, the distance from p to q.

Another car travels the same distance from p to q in the same time of 12 seconds. This car starts from rest at p and accelerates uniformly to its maximum speed of v m/s and then immediately decelerates uniformly to rest at q.

(v) Find v, the maximum speed of this car, giving your answer as a fraction.

(i)



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(ii)
$$v = u + at$$
$$8 = 4 + a(4)$$
$$a = 1 \text{ m/s}^2$$

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(iii)
$$v = u + at$$
$$0 = 8 + a(2)$$
$$a = -4 \text{ m/s}^2$$

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(iv)
$$\text{distance} = \textit{area}$$
$$= 4(4) + \tfrac{1}{2}(4)(4) + 6(8) + \tfrac{1}{2}(2)(8)$$
$$= 16 + 8 + 48 + 8$$
$$= 80$$

10

(v)
$$\text{distance} = \textit{area}$$
$$80 = \tfrac{1}{2}(12)(v)$$
$$80 = 6v$$
$$v = \frac{80}{6} = \frac{40}{3}$$

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2. The velocity of ship A is $3\vec{i} - 4\vec{j}$ m/s and the velocity of ship B is $-2\vec{i} + 8\vec{j}$ m/s.

(i) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the magnitude and direction of the velocity of ship A relative to ship B, giving the direction to the nearest degree.

At a certain instant, ship B is 26 km due east of ship A.

(iii) Show, on a diagram, the positions of ship A and ship B at this instant and show, also, the direction in which ship A is travelling relative to ship B.

(iv) Calculate the shortest distance between the ships, to the nearest km.

(i)

$$V_{AB} = V_A - V_B$$
$$= (3\vec{i} - 4\vec{j}) - (-2\vec{i} + 8\vec{j})$$
$$= 5\vec{i} - 12\vec{j}$$

10

(ii)

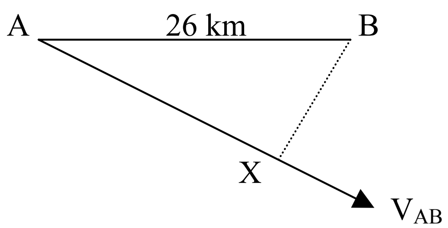
$$\text{magnitude} = \sqrt{(5)^2 + (-12)^2}$$
$$= 13 \text{ m/s}$$

10

$$\text{direction} = \tan^{-1}\left(\frac{12}{5}\right)$$
$$= 67^\circ \text{ south of east}$$

10

(iii)



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(iv) Shortest distance $= |BX|$

$$= 26\sin 67^\circ$$
$$= 24 \text{ km}$$

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3. A particle is projected from a point p on level horizontal ground with an initial speed of 50 m/s at an angle β to the horizontal, where $\tan\beta = \frac{3}{4}$.

(i) Find the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$.

After 4 seconds in flight, the particle hits a target which is above the ground.

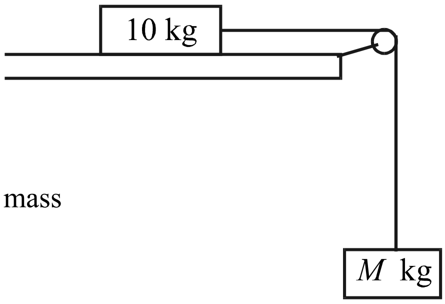
(ii) Show that the distance from the point p to the target is $40\sqrt{17}$ m.

(iii) How far below the highest point reached by the particle is the target?

(iv) Find, correct to the nearest m/s, the speed with which the particle hits the target.

(i)	initial velocity $= 50\cos\beta\ \vec{i} + 50\sin\beta\ \vec{j}$	
	$= 50\left(\frac{4}{5}\right)\vec{i} + 50\left(\frac{3}{5}\right)\vec{j}$	
	$= 40\ \vec{i} + 30\ \vec{j}$	10
(ii)	$r_i = 40(t) = 40(4) = 160$	5
	$r_j = 30.(t) - \frac{1}{2}gt^2$	
	$= 30.(4) - 5(16) = 40$	5
	distance $= \sqrt{160^2 + 40^2}$	
	$= \sqrt{27200} = 40\sqrt{17}$	5
(iii)	At highest point $v_j = 0$	
	$30 - gt = 0$	
	$t = 3$	5
	greatest height $= 30.(t) - \frac{1}{2}gt^2$	
	$= 30(3) - 5(9) = 45$	5
	$\Rightarrow$ target is 5 m below highest point	5
(iv)	$v = 40\ \vec{i} + (30 - gt)\ \vec{j}$	
	$= 40\ \vec{i} - 10\ \vec{j}$	5
	speed $= \sqrt{(40)^2 + (-10)^2} = 41$ m/s	5
		50

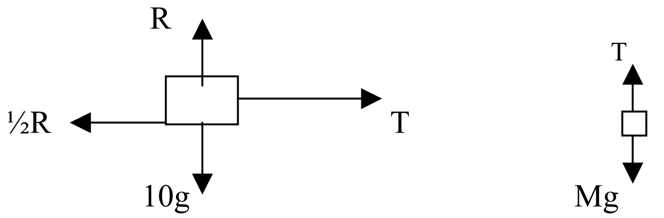
4. **(a)** Two particles, of masses 10 kg and M kg, are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a rough horizontal table.
The coefficient of friction between the 10 kg mass and the table is $\frac{1}{2}$.
The M kg mass hangs freely under gravity.
The particles are released from rest.
The M kg mass moves vertically downwards with an acceleration of 4 m/s^2.



(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the tension in the string.
(iii) Find the value of M.

(b) Calculate the initial speed that a stone must be given to make it skim horizontally across ice so that it comes to rest after skimming 40 m.
The coefficient of friction between the stone and the ice is $\frac{1}{8}$.

(a) **(i)**



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(ii)

$$T - \tfrac{1}{2}R = 10(4)$$
$$T - \tfrac{1}{2}(10g) = 40$$
$$T - 50 = 40$$
$$T = 90$$

10

(iii)

$$Mg - T = M(4)$$
$$10M - 90 = 4M$$
$$\Rightarrow M = 15 \text{ kg}$$

10

(b)

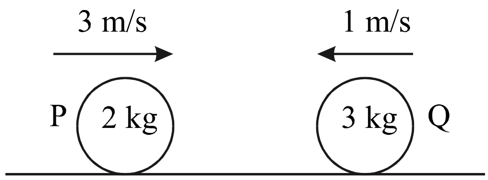
$$\text{Force} = \text{Mass} \times \text{Acceleration}$$
$$-\tfrac{1}{8}(mg) = ma$$
$$\Rightarrow a = -\tfrac{10}{8}$$

10

$$v^2 = u^2 + 2as$$
$$0 = u^2 + 2\left(-\tfrac{10}{8}\right)(40)$$
$$\Rightarrow \quad u = 10 \text{ m/s}$$

10 50

5. A smooth sphere P, of mass 2 kg, moving with a speed of 3 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the opposite direction with a speed of 1 m/s on a smooth horizontal table.
The coefficient of restitution for the collision is *e*.



As a result of the collision, sphere P is brought to rest.

(i) Find the speed of Q after the collision.

(ii) Find the value of *e*.

(iii) Find the fraction of kinetic energy lost due to the collision.

(i) PCM

$$2(3)+3(-1) = 2v_1 + 3v_2$$
$$6-3 = 2(0)+3v_2$$
$$\Rightarrow \quad v_2 = 1$$

15

(ii) NEL

$$v_1 - v_2 = -e(u_1 - u_2)$$
$$0 - 1 = -e(3+1)$$
$$e = \frac{1}{4}$$

15

(iii)

$$\text{KE before} = \tfrac{1}{2}(2)(3)^2 + \tfrac{1}{2}(3)(-1)^2$$
$$= 9 + 1.5$$
$$= 10.5$$

5

$$\text{KE after} = \tfrac{1}{2}(2)(0^2) + \tfrac{1}{2}(3)(1^2)$$
$$= 0 + 1.5$$
$$= 1.5$$

5

$$\text{Loss in KE} = 10.5 - 1.5$$
$$= 9\ \text{J}$$

5

$$\text{Fraction} = \frac{9}{10.5} = \frac{18}{21} \text{ or } \frac{6}{7}$$

5

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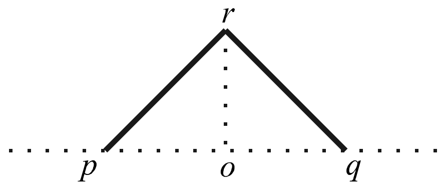
6. **(a)** Particles of weight 2 N, 3 N, 4 N and 1 N are placed at the points (–2, 1), (–1, –1), (2, 2) and (x, y), respectively.
The centre of gravity of the four particles is at the origin.

Find the value of x and the value of y.

(b) Two uniform rods, [rp] and [rq], are rigidly jointed at r. The rods are of equal length. Each rod has a mass of M kg.
$|po| = |oq| = |or| = 0.2$ m.

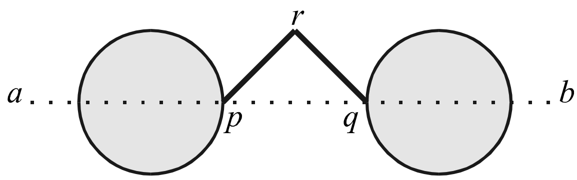


(i) Give a reason why the centre of gravity of the two rods lies on the line or.

(ii) Find the distance of the centre of gravity of the two rods from o.

The diagram shows an optician's advertising sign.

The sign consists of the two rods, [rp] and [rq], described above, now rigidly jointed at p and q to two uniform discs, each of radius 0.2 m and mass M kg.



ab is a horizontal line going through the centre of each disc and the points p and q.
The distance from r to ab is 0.2 m.

(iii) Find the distance of the centre of gravity of the sign from the line ab.

(a)

$$10(0) = 2(-2) + 3(-1) + 4(2) + 1(x)$$
$$x = -1$$ 10

$$10(0) = 2(1) + 3(-1) + 4(2) + 1(y)$$
$$y = -7$$ 10

(b) (i)

$$2M\bar{x} = M(-0.1) + M(0.1)$$
$$\bar{x} = 0$$ 10

(ii)

$$2M\bar{y} = M(0.1) + M(0.1)$$
$$\bar{y} = 0.1$$ 10

(iii)

$$4M\bar{y} = M(0) + 2M(0.1) + M(0)$$
$$4\bar{y} = 0.2$$
$$\bar{y} = 0.05 \text{ m} \quad \text{or} \quad 5 \text{ cm}$$ 10

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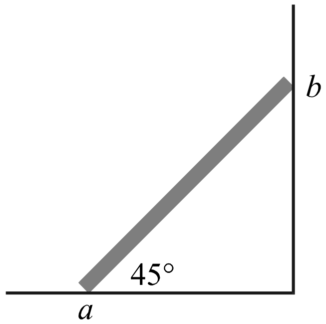
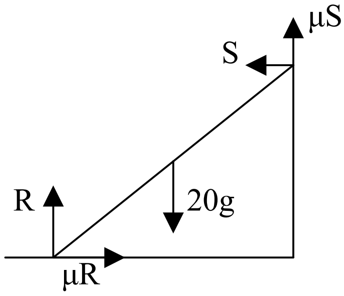
7. A uniform beam, [*ab*], of mass 20 kg and length 8 m, is placed with its end *a* on rough horizontal ground and end *b* against a rough vertical wall.
The coefficient of friction at *a* is μ and at *b* is also μ.
The beam is on the point of slipping when inclined at an angle of 45° to the horizontal.



(i) Show on a diagram all the forces acting on the beam.
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about the point *a*.
(iv) Use the three equations from parts **(ii)** and **(iii)** to show that
$\mu^2 + 2\mu - 1 = 0.$

(i)



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(ii) horiz $S = \mu R$ 5

vert $R + \mu S = 20g$ 5

(iii) Moments about a :

$$S\,(8\sin 45) + \mu S\,(8\cos 45) = 20g(4\cos 45)$$

$$S + \mu S = 100$$ 10

(iv)

$$S = \mu R$$

$$R + \mu S = 200$$

$$S + \mu S = 100$$

from eqs 1 and 2 $R + \mu^2 R = 200$ 5

from eqs 1 and 3 $\mu R + \mu^2 R = 100$ 5

$$R + \mu^2 R = 2(\mu R + \mu^2 R)$$

$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$ 10

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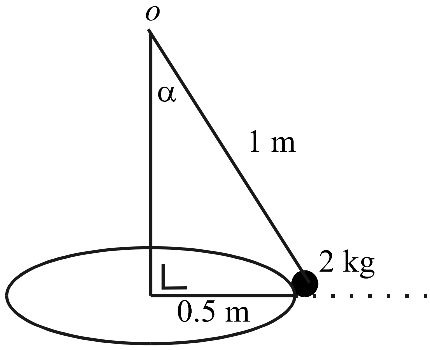
8. **(a)** A vehicle of mass 1000 kg rounds a bend which is in the shape of an arc of a circle of radius 25 m. The coefficient of friction between the tyres and the road is 0.8.

(i) Show on a diagram the three forces acting on the vehicle.

(ii) Calculate the maximum speed with which the vehicle can round the bend without slipping.
Give your answer correct to two places of decimals.

(b) A smooth particle, of mass 2 kg, describes a horizontal circle of radius 0.5 metres on a smooth horizontal table with constant angular velocity 3 radians per second.
The particle is connected by means of a light inelastic string to a fixed point *o* which is vertically above the centre of the circle.
The length of the string is 1 metre.
The inclination of the string to the vertical is α.

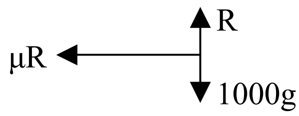


(i) Find α.

(ii) Find the tension in the string.

(iii) Show that the normal reaction between the particle and the table is $20 - 9\sqrt{3}$ N.

(a) (i)



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(ii)

$$\mu R = \frac{mv^2}{r}$$

$$0.8(1000g) = \frac{1000v^2}{25}$$

$$\Rightarrow v = 14.14 \text{ m/s}$$

10

(b) (i)

$$\sin\alpha = \frac{0.5}{1}$$

$$\alpha = 30°$$

10

(ii)

$$T\sin\alpha = mr\omega^2$$

$$T(0.5) = 2(0.5)(3^2)$$

$$T = 18$$

10

(iii)

$$T\cos\alpha + N = 2g$$

$$18\left(\frac{\sqrt{3}}{2}\right) + N = 20$$

$$N = 20 - 9\sqrt{3}$$

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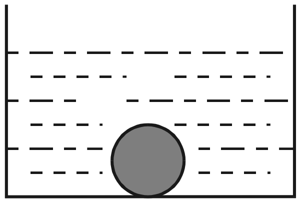
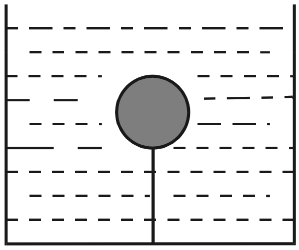
9. A solid sphere of volume V m^3 has a relative density of 1.2.
The sphere is immersed in water in a tank.
The sphere rests on the bottom of the tank.



(i) Show, on a diagram, all the forces acting on the sphere.

(ii) Find, in terms of V, the normal reaction between the bottom of the tank and the sphere.

The sphere is now taken out of the tank of water and placed in a tank of liquid whose relative density is s, where $s > 1.2$.
The sphere is held immersed in the liquid by a light inelastic string tied to the sphere and to the bottom of the tank.

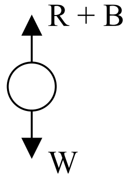


(iii) Explain why the sphere must be tied by a string to the bottom of the tank, so as to remain immersed in the liquid.

(iv) Find the value of s, given that the tension in the string is $1000V$ newtons.

[Density of water = 1000 kg/m^3.]

(i)



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(ii)

$$B = \rho Vg = 1000Vg \quad \text{or} \quad \frac{W(1)}{1.2}$$

10

$$R + 1000Vg = 1200Vg$$
$$R = 200Vg$$
$$= 2000V$$

10

(iii) The sphere must be tied because the density of the liquid is greater than the density of the sphere.

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(iv)

$$T + W = B$$
$$1000V + 1200Vg = 1000(s)Vg$$
$$1300 = 1000(s)$$
$$s = 1.3$$

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